Fourier Optics and Image Analysis

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(Dated: October 15, 2018)

In this experiment we are interested in studying the optical Fourier transform of different set of objects. We show how this technique has strong application at the moment to study an image of its fundamental spacial frequencies and we can transform the objects projection by filtering the spacial frequencies. In this experiment we implement a optical setup that lets us study the image plane and the Fourier plane of an object, understanding how different types objects produce a 2D Fourier transform.

I. INTRODUCTION

In this experiment we are interested in studying the 2D Fourier decomposition of and image threw optical means. Fourier decomposition is an extremely power full tool used threw out many fields. In the current technological environment it is more commonly implemented using computational means. Fourier decomposition in general seance is used to study a signal as the cause of sort of supper position of many fundamental modes or parts, by changing or selecting some of these modes we can alter the over all signal in fundamentally different ways. In signal theory Fourier decomposition is implemented as a way to filter out noise, the selection of information or compression of signals since we can recreate a signal in time based of its fundamental frequencies. In this experiment we want to study an image and its corresponding Fourier decomposition and see what is the physical effect of altering the Fourier decomposition by filtering certain spacial frequencies.

II. THEORETICAL BACKGROUND

The Fourier principle of decomposition states that a signal, for example in time can be reproduced by adding together infinite harmonic components with different frequencies and amplitudes. This is expressed mathematically by and integral over all harmonics as shown in Eq.1

$$f(x,y) = \iint F(\nu_x,\nu_y)e^{-i2\pi(\nu_x x+\nu_y y)}d\nu_x d\nu_y \qquad (1)$$

where $\mathrm{F}(\nu_x,\nu_y) = \mathbf{F}\{f(\nu_x,\nu_y)\}$ is the amplitude for each harmonic. For this transform there is an interesting property when applying the convolution, where the convolution in the conjugate space is the multiplication this is shown in Eq.2.

$$x(t) * h(t) = y(t) \xrightarrow{\mathbf{F}} X(f)H(f) = Y(f) \qquad (2)$$

Now we can also find the inverse Fourier transform, if we have all the information about the frequency decomposition of the signal. We can do this by implementing the inverse operation over the amplitude of the harmonics $f(\nu_x,\nu_y)=\mathbf{F}^{-1}\{F(\nu_x,\nu_y)\}$. This is true in general for any signal, now if we are interested in studying an image and finding the Fourier transform threw optical means we need that diffraction plan is sufficiently in the far-field to be treated as a Fourier transform problem. An other implementation would be using a lens where we we would get the Fourier plane at the foal distance of the lens. Now we are going to discuss the experimental setup used in the experiment to measure observe the Fourier plane and the image plane.

III. EXPERIMENTAL METHOD

The experimental setup implemented is shown as an schematic representation in Fig.1, a HeNe-laser with a wavelength o of 633 nm was used to illuminate different objects that were placed in the object plane. The first two lenses and the spacial filter are used to have a nice light source to illuminate the object, we can observe the two dimensional Fourier transform in the FT-plane where we have access to it and can modify it with filters. With lens L3, an enlarged image of the Fourier plane is projected onto the screen, and L4 gives an enlarged image of the object instead. L 3 can be removed or moved in place of the optical system to observe the Fourier plane or the object plane.

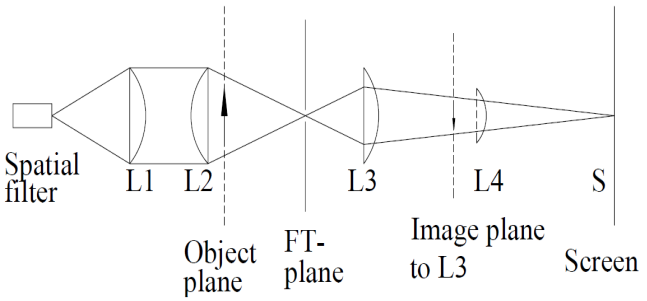


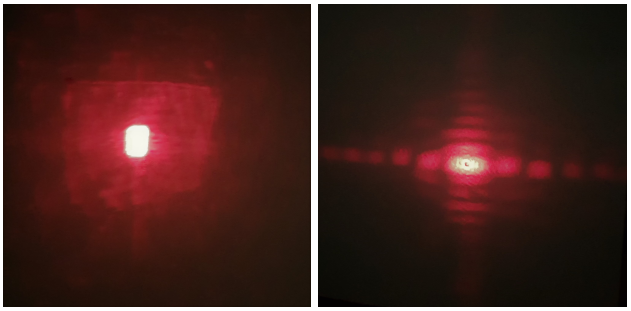
FIG. 1: Experimental setup implemented to study the Fourier transform of different objects. This experimental setup allows us to change between the image plane and the Fourier plane.

A set of different objects were placed and analyzed in

this experiment where we try to understand what is the expected Fourier transform for different set of Objects and how dose the image plane change when we modify the Fourier plane. Now we proceed to the show the recollected images from this experiment where we shoe the object plane and the Fourier plan side by side.

IV. RESULT & DISCCUSION

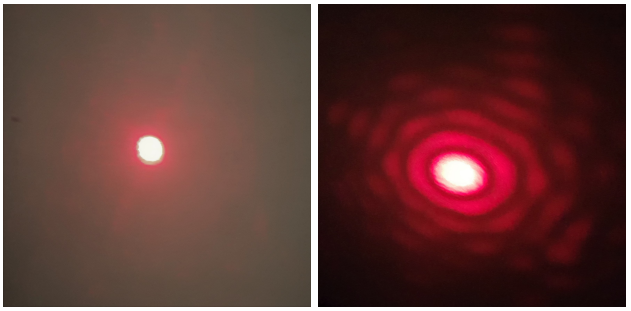
The first object that we studied was a rectangular aperture shown in Fig.2a, where we can that its Fourier transform is a 2D sinc function as expected form Fourier theory. we can see that for bigger aperture in the object plane it has a smaller sinc function on the Fourier plane, since it requires more frequencies to describe a small rectangular function.



(a) Image plane (b) Fourier plane

FIG. 2: Rectangular aperture giving a 2d sinc function as its Fourier transform.

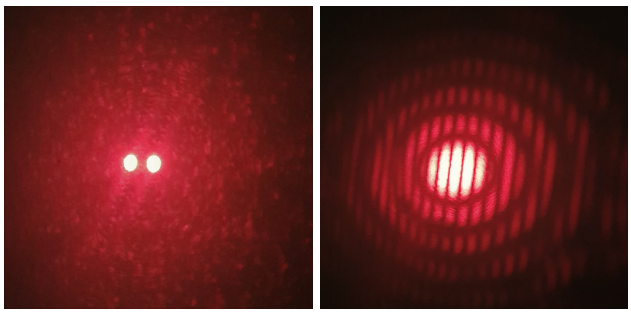
We change the object with a circular aperture as shown in Fig.3a and as expected we get an Ariy pattern as its Fourier transform shown in Fig.3b. where we know theoretically similarly to the rectangular case the smaller the aperture the more spacial frequencies are going to be needed to describe the object.



(a) Image plane (b) Fourier plane

FIG. 3: Circular aperture used as an object, producing and Ariy pattern as its Fourier transform.

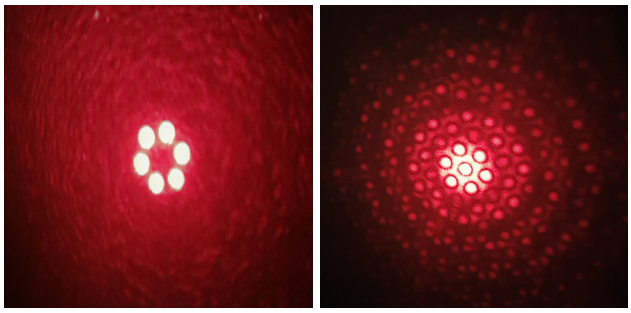
Following the circular aperture we studied the multiplication between the previous system and a set of translated delta functions Fig.4a. Keeping in mind what was discussed in the previews section about the properties of convolution, we can see in Fig.4b that we get the convolution of the two expected Fourier transforms an Ariy pattern as shown previously in Fig.3b and an interference pattern or it can also be thought as if we had a regular grid that was convoluted with a rectangle thus, producing a Fourier transform as a interference pattern.



(a) Image plane (b) Fourier plane

FIG. 4: The object is a multiplication of a set of translated delta functions and a curricular aperture giving of a Fourier transform that is the convolution of the individual expected transforms.

With this insight we study a similar system that composed of more delta functions, we can consider the object shown in 5a as the multiplication of three simple objects, a regular grid in a hexagonal form, a large circular aperture, that selects a single hexagonal shape, and a small circular aperture that is set over each of the delta functions. From this interpretation we can expect the convolution of this three objects as is seen in Fig.5b where we can see a large Ariy pattern for the small aperture and a regular grid with small Ariy pattern supper imposed due to the large circular aperture.

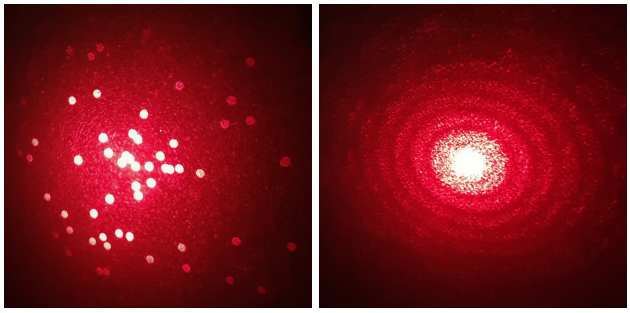


(a) Image plane (b) Fourier plane

FIG. 5: The object consist of three supper imposed systems a small aperture a regular grid with a hexagonal pattern and a large aperture producing the convolution of these three systems.

Now we want to study what happens if we have a grid that is not regular, what type of pattern would we see,

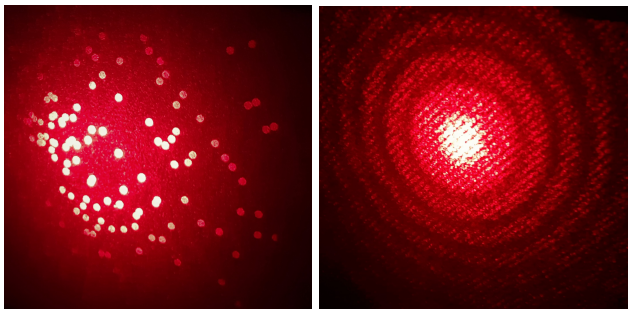
this object is shown in Fig.6a where we multiply it i=with a circular aperture. We can see from its Fourier transform Fig.6b that it is a random grid since we don't see an underlying interference pattern and only a general Ariy pattern from the circular pattern.



(a) Image plane (b) Fourier plane

FIG. 6: Random grid multiplied with a circular aperture giving off an over all Ariy pattern with no interference pattern produce by the grid since its has a random order.

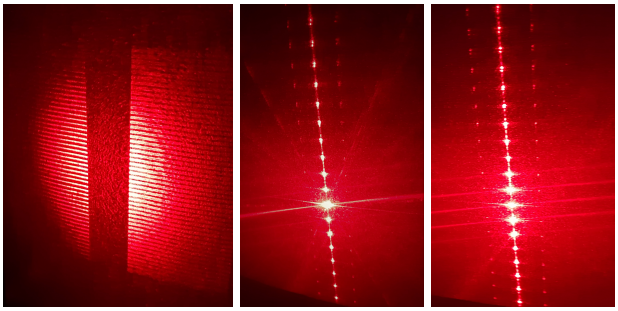
In Fig.7a we have taken a double exposure of the random grid shown in Fig.6a, If we study its Fourier transform Fig.7b now we can see that its similar to the Fourier transform in Fig.6b but know we can tell that there is an emerging interference pattern similar to the one seen from the regular grid in Fig.4b. This happens because the object is a double exposure of random grid and this produces a regularity in this new object, thus allowing the emergence of an interference pattern.



(a) Image plane (b) Fourier plane

FIG. 7: Double exposure of a random grid multiplied with a circular aperture giving off an over all Ariy pattern with an interference pattern produce by the double exposure of the random grid.

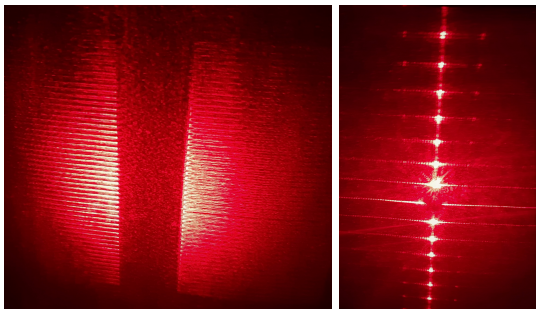
The next object studied was a set of gratings that have different spacing between lines as can be seen in Fig.8a from this set of gratings we get two different Fourier transform that are what would be expected in general the different orders of diffraction Fig.8b and Fig.8c respectively for each grating. Th main difference between the two gratings transform can be seen that is the intensity from the zeroth order or the middle peak as can be seen from Fig.8b and Fig.8c. Now we could try to see what is the effect of filtering out the zeroth order from the large spacing grid (right in Fig.8a).



(a) Image plane (b) Fourier plane (c) Fourier plane

FIG. 8: A set of grating with different spacing between lines are used as the object, and we look at the different orders of diffraction in the Fourier plane where the main difference is the intensity in the zeroth order.

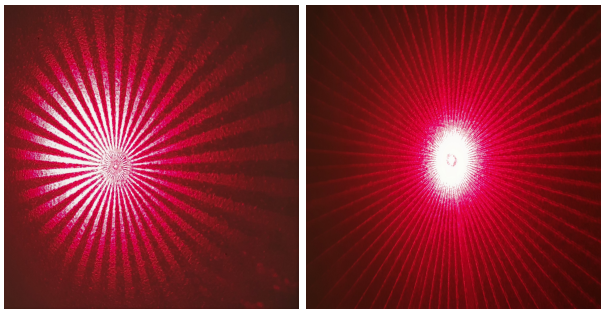
We cover the zeroth order of the right grating from Fig.8a as is shown in Fig.9b and we look at the image plan in Fig.9a and we can see that the spacing of the right grating changes making it the same as the left one Fig.9a. This tells us that the original gratings are just the inverse of one an other the space between lines from on if is the width of the lines for the other. When we filter the zeroth order we take away the mean value of the grating thus changing the over all pattern allowing us to see how the spacing is related between both gratings.



(a) Image plane (b) Fourier plane

FIG. 9: A set of grating with the same spacing between lines were produce by filtering of the zeroth order of the right grating as shown in b).

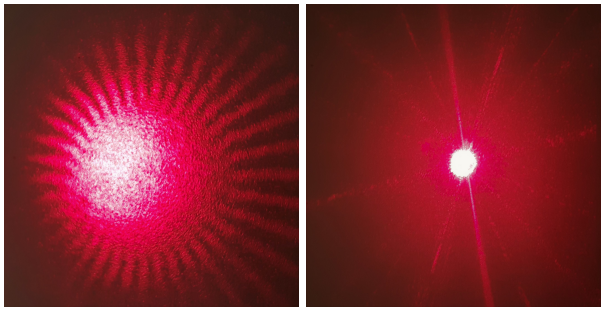
The next object studied was a sector star, to this system we implemented multiple spacial filters to the Fourier transform to observe what were the outcome over the image plane. In Fig.10 we see the original object image with its corresponding Fourier transform. Now we apply filters in the Fourier plane.



(a) Image plane (b) Fourier plane

FIG. 10: Sector star with its corresponding Fourier transform

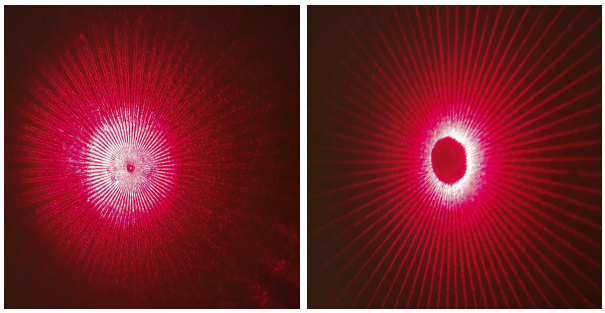
First we select only the low spacial frequencies as is shown in Fig.11b and we observe in the image plane that the image loses all the details in the image compared with Fig.10a, we can still see the general tendency but not the finer details.



(a) Image plane (b) Fourier plane

FIG. 11: Sector star with a high spacial frequencies filter as shown in Fourier transform producing the image plane

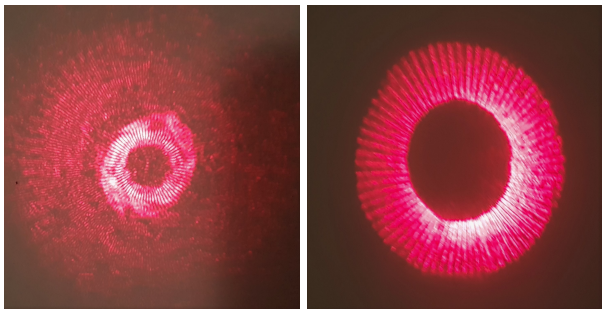
The next measurement would be to do the inverse process from the previous filtering letting only the high spacial frequencies pass from the Fourier transform as is shown in Fig.12b. From this selection we observe the image plane in Fig.12 where we can see now the opposite effect on the sector star, we maintaining all the smaller details in general but we lose all the large defining traits as the filling in each of the spacing of the sector star.



(a) Image plane (b) Fourier plane

FIG. 12: Sector star with a low spacial frequencies filter as shown in Fourier transform producing the image plane

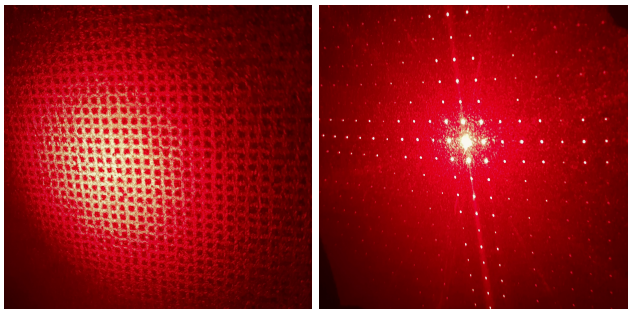
Now it would be interested in applying a combination of the two previous cases by applying a band pass to the Fourier transform as can be seen in Fig.13b, giving off its corresponding image Fig.13 where we can see that there is still an resemblance to a sector star but we have lost most of the information to actually produce a accurate image.



(a) Image plane (b) Fourier plane

FIG. 13: Sector star with a band pass spacial frequencies filter as shown in Fourier transform producing the image plane

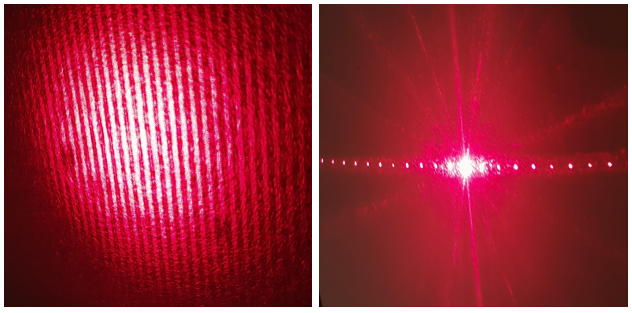
The next object we study is a cross grating as shown in Fig.14a and it has a predictable Fourier transform Fig.14b since we would expect the different orders of diffraction for both axis's. One may wonder if we filter ans select only on of the diffraction lines in the Fourier plane creating the Fourier transform for normal grating what would we see in the image plane.



(a) Image plane (b) Fourier plane

FIG. 14: Cross grating with its corresponding Fourier transform the different orders of diffraction

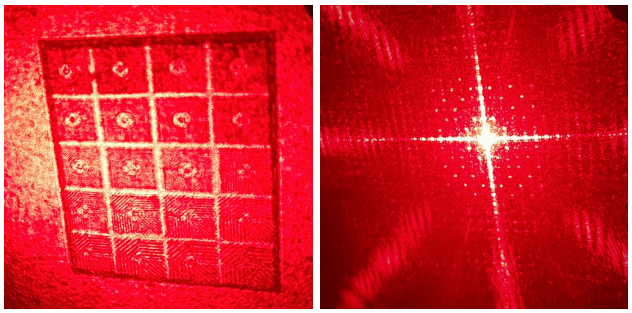
Selecting a single order of diffraction as can be seen in Fig.15b with this filtering we can see that in the image plane Fig.15a we see a normal grating as it would be expected.



(a) Image plane (b) Fourier plane

FIG. 15: Cross grating filtered with only on set horizontal set of diffraction in the Fourier transform making it a normal grating

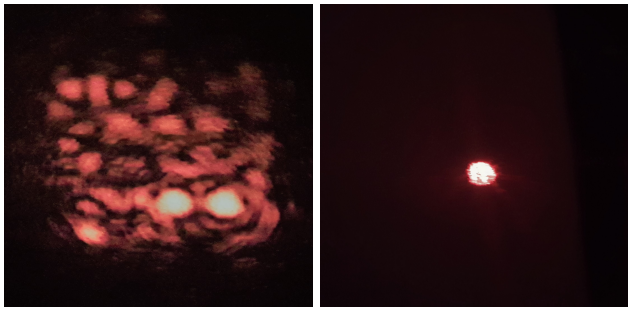
Now we want to study a system that has multiple gratings all in different directions and spacing shown in Fig.16a, this object produce a Fourier transform with many diffraction order in all directions since it contains a large set of gratings superimposed Fig.16b and we can see a stronger set of lines orthogonal to each other pacing to the middle this as brighter because the contain the information of ht outline of the object.



(a) Image plane (b) Fourier plane

FIG. 16: Object with multiple order of diffraction gratings producing a complex Fourier transform

If we filter selecting a set of spacial frequencies Fig.17b what we observe in the image plane Fig.17a is only the components of the image the contains this set of frequencies thus letting us understand what type of gratings are inside of the object by selecting a certain set of frequencies.



(a) Image plane (b) Fourier plane

FIG. 17: Object with multiple order of diffraction gratings showing only certain parts of the object with the corresponding frequencies selected from the complex Fourier transform

Now the last system we studied was a set of gratings that intercepted with each other Fig.18 and then we illuminated it with white light to see what type of Fourier transform it gives off. We also selected a set of three colors of the transform and observed how the image plane changed.

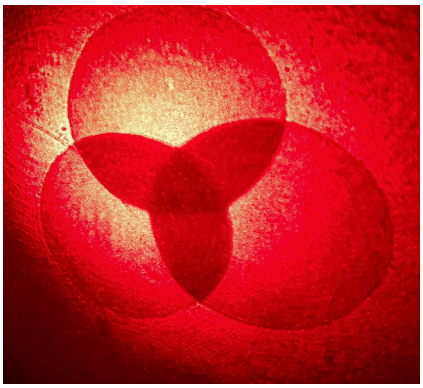


FIG. 18: Three big circular areas with different gratings

When we illuminate with white light we observe as expected a similar transform Fig.19aas we have seen previously for other grating and by selecting a set of colors from this transform we can change the image plane to observe what color is being diffracted by which grating as can be seen in Fig.19b



(a) Fourier plane (b) Image plane

FIG. 19: White light illuminated of the three big circular areas with different gratings producing the Fourier transform and by selecting three colors we get the image plane

From this experiment we can see exactly how strong is this technique since it give us a tool to study the composition in frequencies do different objects just by selecting frequencies in the Fourier plane. Of this experiment we can see how the Fourier transform is such a strong technique in so many fields sin it lets us understand a complex system as part of a smaller set of underlying systems.

[1] Optics and Optical Design (FAFF01), "Fourier Optics and Image Analysis", laboratory guide 2017.

[2] Le Nguyen Binh, Advanced Digital Optical Communications, Second Edition, 2nd Edition.