Helium-Neon Laser

Frans Mattsson
Sergio Gutiérrez

Supervisor: Jasper Peschel

Lab performed: Dec 4, 2018

Contents

1 Introduction **1**

2 Theory **1**

3 Preparation exercises **2**

4 Method **5**

5 Results and discussion **6**

6 Conclusions **10**

References **11**

1 Introduction

Light sources utilising Light Amplification By Stimulated Emission of Radiation (LASER) are ubiquitous in today's society, being used in communication, medicine and industrial production to mention a few. The fundamental parts of a laser are the gain medium, the pump and the resonator, all of which are required for an operational system. This lab exercise will be focused around understanding the fundamental workings of a He-Ne (Helium-Neon) laser. We will acquire a deeper understanding of the modes that exist in a laser cavity and the stability of the lasing process in the cavity. This is going to be done by executing a set of measurements inside and outside of the resonator. We also compare our measurements to the theoretical models.

2 Theory

A mirror cavity, in the ray optics picture, is said to be stable if rays remain confined indefinitely inside of it. Through a derivation using matrix formality for ray optics [1], it can be shown that a cavity is stable when the radii of curvature, R_1 and R_2, of the mirrors and the distance d between them conform to the inequality

$$0 \leq g_1 \cdot g_2 \leq 1, \quad g_1 \cdot g_2 = \left(1 + \frac{d}{R_1}\right)\left(1 + \frac{d}{R_2}\right). \tag{1}$$

In order for light of a certain frequency to achieve lasing in a laser cavity, it has to fulfil certain requirements, the two most important of which are that the frequency matches a mode of the cavity and that the gain for the frequency exceeds the losses in the cavity. For a longitudinal mode of a plane wave the resonance condition on its frequency is simply

$$\nu_q = q\frac{c_0}{2n_i d}, \quad q = 1, 2, 3... \tag{2}$$

where q is the mode index, c_0 is the speed of light in vacuum, n_i is the refractive index inside the cavity and d is the distance between the mirrors. This can conveniently be expressed in terms of the free spectral range of the cavity ν_F as

$$\nu_q = q\frac{c_0}{2n_i d} = q\nu_F, \quad q = 1, 2, 3... \tag{3}$$

The restriction originates from the requirement of the distance being an integer multiple of half of the wavelength of the light $d = q\frac{\lambda}{2}$. The plane wave picture is, however, not applicable to real cavities. Instead, modes of the cavity are taken to be Gaussian beams which introduces the Gouy phase into the picture. Going further, the Hermite-Gaussian modes can be used, the most basic case of which is the Gaussian beam. The Hermite-Gaussian modes introduce periodicity also in the transverse direction having the resonant frequencies

$$\nu_{q,l,m} = q\nu_F + (l + m + 1)\frac{\Delta\zeta}{\pi}\nu_F, \quad q = 1, 2, 3..., \quad l, m = 0, 1, 2, 3... \tag{4}$$

where l and m are the transverse indices of the mode, $\Delta\zeta = tan^{-1}(z_1/z_0) - tan^{-1}(z_2/z_0)$ where z_1 and z_2 are the positions of the mirrors relative to the beam waist and z_0 is the Rayleigh length.

In a cavity it is of interest to have some manner of measurement of how much a beam bounces inside the cavity before it leaks out or is absorbed by imperfections. This can be quantified by the finesse of a cavity, described by Eq.5, which depends on the attenuation factor $h = |r|e^{-i\phi}$. If there are no losses inside of the cavity only those given by the mirrors $|r|^2 = R_1 R_2$ where R_1 and R_2 are the power reflectances of the two mirrors respectively.

$$F = \frac{\pi\sqrt{|r|}}{1-|r|} \tag{5}$$

The electric field of a Hermite-Gaussian mode in a cavity can be written as

$$E_{l,m}(x,y,z) = A_{l,m}\frac{W_0}{W(z)} \cdot H_l\left(\sqrt{2}\frac{x}{W(z)}\right) \cdot H_m\left(\sqrt{2}\frac{y}{W(z)}\right) e^{-\frac{x^2+y^2}{W(z)^2}} e^{-j\left(\frac{k(x^2+y^2)}{2R(z)} - (1+l+m)\zeta\right)} \tag{6}$$

where $A_{l,m}$ is the amplitude of the mode, H_i are the Hermite polynomials of order i, W_0 is the beam waist, $W(z)$ is the beam radius and R is the radius of curvature of the beam. The intensity of this electrical field distribution then becomes

$$I_{l,m}(x,y,z) = |A_{l,m}|^2 \left| \frac{W_0}{W(z)} \cdot H_l\left(\sqrt{2}\frac{x}{W(z)}\right) \cdot H_m\left(\sqrt{2}\frac{y}{W(z)}\right) e^{-\frac{x^2+y^2}{W(z)^2}} \right|^2 \tag{7}$$

The power consumption in a resistive component in an electric circuit can be calculated as

$$P = U \cdot I, \tag{8}$$

where U is the voltage over the component and I is the current running through it. The energy efficiency of a laser is defined as the quotient of the power of the output light and the input power as

$$\eta_{laser} = \frac{\text{laser power}}{\text{input power}} \tag{9}$$

For a photo diode, a figure of interest is the so called quantum efficiency measured as the fraction of photons that generate a photo-electron

$$\eta_{diode} = \frac{\text{number of photo-electrons}}{\text{number of incoming electrons}}. \tag{10}$$

It is also worth to mention how a single mode can be selected in a cavity. The most common way is using a Fabry-Perot interferometer also known as a Fabry-Perot etalon. A direct way of understanding mode selection is by looking at the comparison of the respective free spectral ranges of the cavity and the etalon. The free spectral range given in Eq.3 and given that an etalon is much smaller than the cavity (since it is an element set inside of a cavity) then $\nu_{Fc} \ll \nu_{Fe}$. Since modes need to match both the mode frequencies of the cavity and the transmission frequencies of the etalon, these conditions can be matched to produce a single mode laser.

3 Preparation exercises

3.1

Using Eq.8

$$P = UI = 1450\,\text{V} \cdot 6.5\,\text{mA} = 9.4\,\text{W} \tag{11}$$

3.2

We have

$$\frac{P}{h\nu} = \phi \left[\frac{photons}{s}\right]. \tag{12}$$

Quantum efficiency (from Eq.10) $\eta_{diode} = 0.5 \implies$

$$I = 0.5\frac{P}{h\nu}e = 0.5 \cdot \frac{9.425\,\mathrm{W}}{h\nu}e = 0.256\,\mathrm{mA}, \tag{13}$$

where h is Planck's constant, $\nu = c/\lambda$ and e is the elementary charge.

3.3

We have the stability condition

$$0 \leq g_1 \cdot g_2 \leq 1 \tag{14}$$

and as defined in the problem,

$$g_1 = 1, \quad g_2 = 1 + \frac{d}{-0.45} \tag{15}$$

The values of d that retain $g_1 \cdot g_2$ within the stability region can easily be seen to be $d \in [0\,\mathrm{m}, 0.45\,\mathrm{m}]$

3.4

We have the mode frequencies for the Hermite-Gaussian modes in a resonator

$$\nu_{q,l,m} = q\nu_F + (l + m + 1)\frac{\Delta\zeta}{\pi}\nu_F, \quad q = 1, 2, 3..., \quad l, m = 0, 1, 2, 3... \tag{16}$$

Here $\Delta\zeta$ is determined by

$$\Delta\zeta = atan\left(\frac{z_2}{z_0}\right) - atan\left(\frac{z_1}{z_0}\right) = 0.49\,\mathrm{m}. \tag{17}$$

The difference between the two modes then becomes

$$\nu_{00q} - \nu_{20q} = 2 \cdot \frac{\Delta\zeta}{\pi}\nu_F = 163\,\mathrm{MHz}. \tag{18}$$

3.5

We are given a wavelength of 633 nm and the definitions for the resolving power $\mathcal{R}$ of a cavity is given by

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = \frac{\pi}{2}m\sqrt{F}\ , \ m_{max} = \frac{2d}{\lambda} \rightarrow \mathcal{R} = \frac{\pi}{2}\frac{2d}{\lambda}\sqrt{\frac{\pi\sqrt{|r|}}{1 - |r|}} \tag{19}$$

for the first part we have a 1 mm glass plate and we have can find the reflectance using the Fresnel equation for normal incidence given by

$$r = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| = 0.2 \rightarrow \mathcal{R} = \pi \frac{1\,\text{mm}}{633\,\text{nm}} \sqrt{\frac{\pi\sqrt{|0.2|}}{1 - |0.2|}} = 6 \cdot 10^3$$

For the second case when we have a 5cm etalon with $r = 0.85$ we get a resolving power of:

$$\mathcal{R} = \pi \frac{5\,\text{cm}}{633\,\text{nm}} \sqrt{\frac{\pi\sqrt{|0.85|}}{1 - |0.85|}} = 1.1 \cdot 10^6 \tag{20}$$

3.6

We measure with a Fabry-Perot interferometer the free spectral range of a laser as shown in Fig.1. With this we can determine the cavity length and the radius of curvature of the mirrors used.

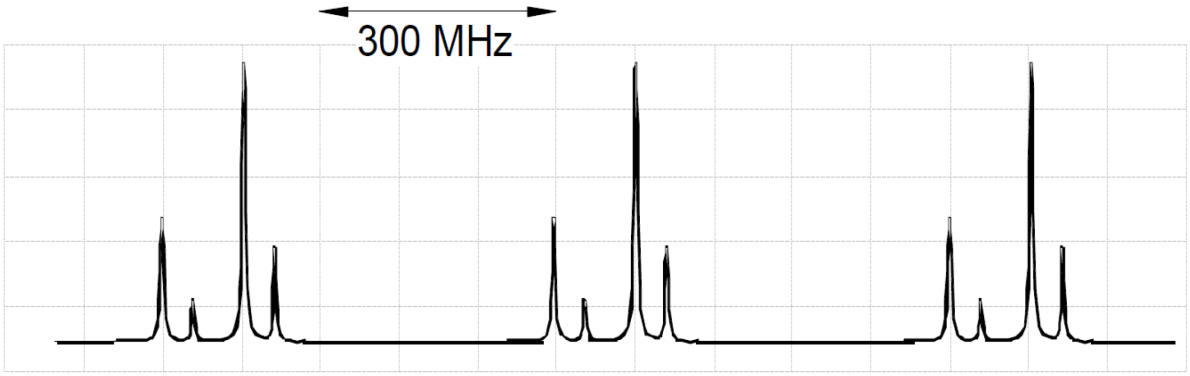


Figure 1: Oscilloscope picture from a measurement of the mode frequencies using a Fabry-Perot interferometer.

The pattern with four peaks repeats itself three times, an effect which is caused by more than one transmission order of the etalon overlapping in frequency with the modes of the laser. Within each pattern of four peaks, the most intense peaks are thought to be the purely longitudinal modes whereas the less intense peaks correspond to transverse modes but with the same q-value as the longitudinal modes. In order to determine the cavity length, we can use the free spectral range for the longitudinal modes

$$\nu_F = 100\,\text{MHz} = \frac{c_0}{2d} \rightarrow d = \frac{c_0}{2 \cdot 100\,\text{MHz}} = 1.5\,\text{m} \tag{21}$$

Now to find the radius we can try to calculate the radius of curvature of the mirrors used by assuming $z_2 = -z_1 = d/2 = 0.75m$ and taking the two consecutive transverse modes with frequency difference $\Delta\nu = 40\,\text{MHz}$ we can use Eq.16 giving a z_0 and radius of curvature at the mirrors R as

$$z_0 = \frac{d}{2 \cdot \tan\left(\frac{\pi\Delta\nu}{2\Delta\nu_F}\right)} = 1.03\,\text{m} \tag{22}$$

$$R = \left|z_1 + \frac{z_0^2}{z_1}\right| = 2.2\,\text{m} \tag{23}$$

4 Method

A 633 nm Helium-Neon laser was disassembled and realigned using an external alignment laser. This laser scheme is described in the energy diagram shown in Fig.2 giving off light with wavelength 543 nm , 632.8 nm. The intensity of the laser was measured with a Milli-ampere meter connected to a photo diode with which we optimized the position of the adjustable mirror of the laser cavity. The conversion factor used between the current in the Milli-ampere meter and the power of the incoming light was 0.4 mA/mW.

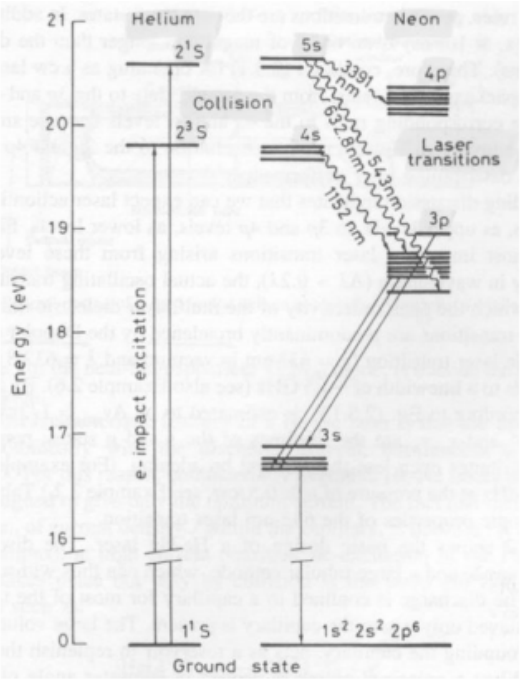


Figure 2: Energy levels for the He-Ne laser[1]

When the laser was properly setup we could execute different sets of measurements, inside and outside of the cavity to understand the relation of the power and the reflectively of the mirrors. The setup that was implemented for the He-Ne laser without taking into account the external elements used for alignment is roughly shown in Fig.3 where we had access to one of the mirrors of the cavity and could change it at will. The fixed mirror in the cavity was a plane mirror and this was not changed throughout all of the measurements done.

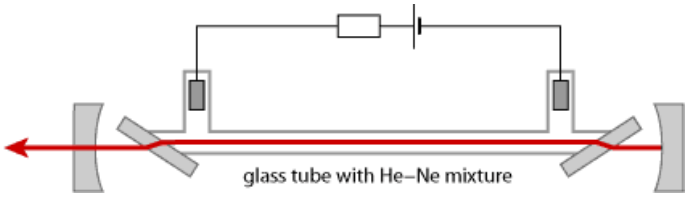


Figure 3: Experimental setup used for the He-Ne laser showing the tube and the cavity.

With this experimental setup we proceed to execute a set of different measurements inside and outside the cavity, also changing the length of it and the mirrors giving us a wide range of possibilities to study this resonator system.

5 Results and discussion

When the loose mirror of the laser cavity was adjusted to attain maximum output power, the final current in the Milli-ampere meter was 505 mA. Using the conversion factor 0.4 mA/mW, gives the power

$$P_{laser} = \frac{0.505\,\text{mA}}{0.4\,\text{mA/mW}} = 1.26\,\text{mW} \tag{24}$$

Using Eq.8 and the values provided in the lab exercise instructions [2], the input power to the He-Ne laser was

$$P = U \cdot I = 1.45\,\text{kV} \cdot 6.5\,\text{mA} = 9.425\,\text{W}. \tag{25}$$

The efficiency of the laser, from Eq.9, then becomes

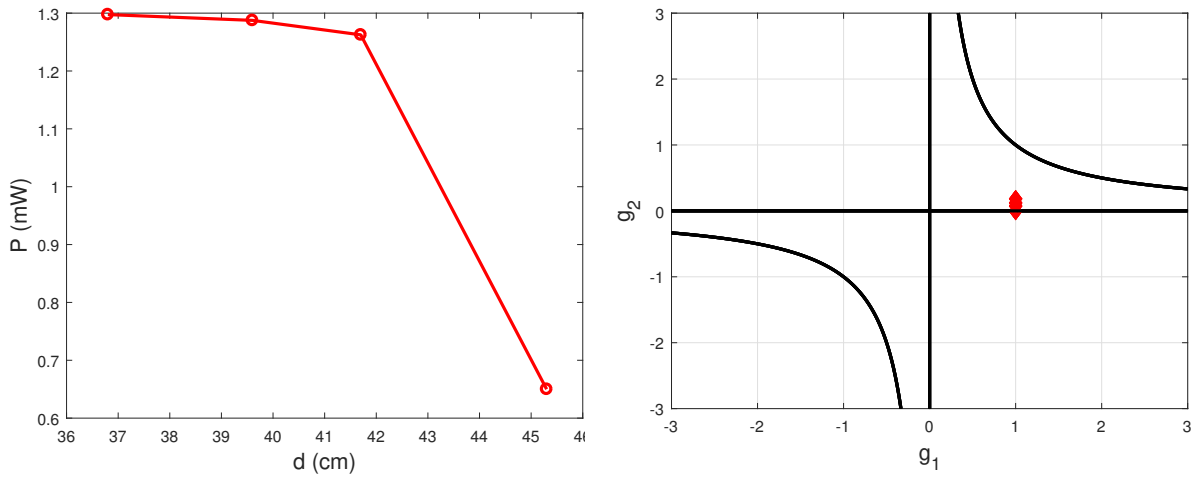
$$\eta_{laser} = \frac{\text{laser power}}{\text{input power}} = \frac{1.26\,\text{mW}}{9.425\,\text{W}} = 0.013\% \tag{26}$$

From the conversion factor of the photo diode, 0.4 mA/mW, and the approximate wavelength of the laser light, 633 nm, the quantum efficiency of the diode from Eq.10 was

$$\eta_{diode} = \frac{\text{number of photo-electrons}}{\text{number of incoming electrons}} = \frac{I_{diode}/e}{P_{laser}/\frac{hc}{\lambda}} \tag{27}$$

$$= \frac{(0.505\,\text{mC/s})/(1.6 \times 10^{-19}\,\text{C})}{(1.26\,\text{mJ/s})/\left(\frac{6.63 \times 10^{-34}\,\text{Js} \cdot 3 \times 10^{8}\,\text{m/s}}{633 \times 10^{-9}\,\text{m}}\right)} = 0.78 \tag{28}$$

The laser power was optimized by slightly rotating the mirror using knobs for a few different resonator lengths. The results are shown in Fig.4a with the points inserted into the stability diagram in Fig.4b. As the resonator approaches the edge of its stability region, the intensity decreases for the same input power. For longer cavity lengths, there was no lasing at all. Other effects that affect the laser power are the intensity of the pumping and the reflectivity of the mirrors used in the cavity. In our case we could not change the intensity of the pumping and as was discussed previously we have a cavity consisting of a permanent planar mirror and a loose spherical mirror where only the loose one could be changed.



(a) (b)

Figure 4: (a) The laser power for a few different cavity lengths. (b) The points in the stability diagram corresponding to the points in (a).

In the setup, there were Brewster windows on both sides of the cavity with the purpose of polarizing the laser beam. The Brewster window, adjusted so that the laser beam would hit the surface at the Brewster angle, eliminates reflection for the p-polarized light. As a consequence, the s-polarized light which is partially reflected at the window will be dominated by the p-polarized light inside the cavity leading to mostly p-polarized light coming out from the cavity. When the polarization of the output from the laser was transmitted through a linear polarization filter this was confirmed.

A hair cross was inserted into the cavity and was adjusted to block some regions of the beam. This led to some modes, occupying this volume inside the cavity, being attenuated, giving preference to other modes that could be seen by projecting the laser beam onto a screen in the far field. When blocking the most intense mode TEM_{q00} this allows us to see the less intense modes that are oscillating inside the cavity. With this setup, the TEM_{q10} and TEM_{q01} could be isolated by intercepting the beam with the vertical and horizontal hair of the cross hair respectively. To approximate what thickness l of the hair would be needed to give preference to the TEM_{q10} mode over the TEM_{q10} mode, the condition is set to be that the intensity ratio of the two modes be less than 1% at the edge of the hair

$$\frac{I_{q01}(l/2)}{I_{q00}(l/2)} < 0.01. \tag{29}$$

where l is the thickness of the hair centred around the beam centre. Using the corresponding Hermite polynomials for the two modes, $H_0(x) = 1$, $H_1(x) = 2x$, and cancelling all factors common to both intensities from Eq.7, the ratio becomes

$$\frac{I_{q01}(l/2)}{I_{q00}(l/2)} = \frac{4(\frac{\sqrt{2}}{W}\frac{l}{2})^2}{1} = \frac{2 \cdot l^2}{(0.4\,\mathrm{mm})^2} = l^2 \cdot 9.45 \times 10^6. \tag{30}$$

Here, the beam waist W was calculated using Eq.3.1-27 and 3.1-8 from Saleh and Teich [1] with $d = 0.43$, $R_1 = -\infty$ and $R_2 = 0.45\,\mathrm{m}$. The position in the cavity was taken to be 30 cm from the permanent mirror. This conforms to the set condition for

$$l^2 \cdot 9.45 \times 10^6 < 0.01 \iff l < 32\,\mu\mathrm{m} \tag{31}$$

A scanning Fabry Perot interferometer was mounted outside the cavity in the beam's path and the transmission frequency was scanned by alternating the voltage over the piezoelectric crystals connecting the two mirrors making up the interferometer. The contraction and expansion of the piezoelectric crystal brought about a change in the interferometer length and thus its transmission frequencies. By using a saw tooth pulse for the applied voltage, the frequency content of the laser could be observed as a function of time on the screen of the oscilloscope. The same pattern was repeated a few times on every pass of the saw tooth as new constructive interference orders, and therefore transmission peaks, of the Fabry Perot etalon overlapped with the laser spectrum. The frequency scanning was assumed to be linear and with the known free spectral range of the etalon given as 2 GHz, the scaling factor between differences in time and frequency was calculated by measuring the time required for the same pattern to repeat itself

$$k_s = \frac{\Delta\nu}{\Delta t} = \frac{2\,\mathrm{GHz}}{680\,\mathrm{ms}} = 2.94 \times 10^9\,\mathrm{Hz/s}. \tag{32}$$

The frequency difference between adjacent longitudinal modes could then be calculated using this conversion factor as

$$\Delta\nu_l = \Delta t \cdot k_s = 54\,\mathrm{ms} \cdot 2.94 \times 10^9\,\mathrm{Hz/s} = 160\,\mathrm{MHz}. \tag{33}$$

The same was done for the transverse modes that were visible close to their respective longitudinal mode giving

$$\Delta\nu_t = \Delta t \cdot k_s = 20\,\mathrm{ms} \cdot 2.94 \times 10^9\,\mathrm{Hz/s} = 59\,\mathrm{MHz} \tag{34}$$

It is unsurprising that the spacing of the transverse modes is greater than that of the longitudinal modes. The spacing can be readily deduced from Eq.4 giving

$$\Delta\nu_l = \nu_F = \frac{c}{2d} \tag{35}$$

$$\Delta\nu_t = \frac{\Delta\zeta}{\pi}\nu_F = \frac{\Delta\zeta}{\pi}\frac{c}{2d}. \tag{36}$$

where $\Delta\zeta \leq \pi$ leading to $\Delta\nu_t < \Delta\nu_l$. With this knowledge it is possible to calculate the cavity length from the frequency spacing as

$$\Delta\nu_l = \frac{c}{2d} \quad \Longleftrightarrow \quad d = \frac{c}{2\Delta\nu_l} = \frac{3 \times 10^8\,\mathrm{m/s}}{2 \cdot 160\,\mathrm{MHz}} = 94\,\mathrm{cm}. \tag{37}$$

The measured length was 43 cm which differs from the calculated value by close to a factor of 2. If the measurement apparatus is somewhat decent and the theory is correct, this large of a discrepancy would not be expected. This leads to a suspicion that there is either some error in the calculations or with the measurement apparatus.

A Fabry Perot etalon was inserted into the cavity in addition to the one already present outside the cavity for measuring purposes. The etalon's presence inside the cavity leads to further restrictions on which frequencies can be present. They have to match both a mode of the cavity but also a transmission order of the etalon. In the case that the width of the transmission orders of the etalon are no wider than the frequency difference between consecutive modes in the cavity, this will give rise to a single mode laser.

Next we are interested in measuring the reflectively of the mirrors used. First the loose mirror was adjusted for maximal output power and the power transmitted through

both the loose and the permanent mirrors was measured. With a known reflectivity for the loose mirror of $R_1 = 0.983$ and assuming that there is no transmission through the other mirror, the power inside the cavity could be approximated. The standing waves inside of the cavity can be thought of as counter-propagating waves and the output power, P_{out}, of the laser should then be

$$P_{out} = \frac{(1-R)P_{cavity}}{2} \tag{38}$$

where P_{cavity} is the power in the cavity and R is the reflectivity of the out-coupling mirror. This gives the following expression for the cavity power and inserting the values measured for the loose mirror the power becomes

$$P_{cavity} = \frac{2P_{out}}{(1-R)} = \frac{2 \cdot 0.85\,\mathrm{mW}}{(1-0.983)} = 100\,\mathrm{mW} \tag{39}$$

With known cavity power, the reflectivity of the permanent mirror, R_p, could also be approximated using Eq.38. The power transmitted through the mirror was measured to be 0.1 mW giving a reflectance of

$$P_{out} = \frac{(1-R_p)P_{cavity}}{2} \quad \Longleftrightarrow \tag{40}$$

$$R_p = 1 - 2 \cdot \frac{P_{out}}{P_{cavity}} = 1 - 2 \cdot \frac{0.1\,\mathrm{mW}}{100\,\mathrm{mW}} = 0.998 \tag{41}$$

This reflectivity is expected to be higher than that of the loose mirror as is indeed the case.

The loose mirror was replaced by a mirror with longer focal length and the power being transmitted through the mirror was once again measured. The transmitted power through the permanent mirror was also measured. The reflectivity of the second loose mirror could then be calculated in the same way as was done for the permanent mirror previously. The new cavity power could be calculated using the power transmitted through permanent mirror with now known reflectivity as

$$P_{cavity} = \frac{2P_{out}}{(1-R)} = \frac{2 \cdot 0.138\,\mathrm{mW}}{(1-0.998)} = 138\,\mathrm{mW}, \tag{42}$$

and using the power transmitted through the loose mirror

$$R_2 = 1 - 2 \cdot \frac{P_{out}}{P_{cavity}} = 1 - 2 \cdot \frac{0.0625\,\mathrm{mW}}{138\,\mathrm{mW}} = 0.9991 \tag{43}$$

where P_{out} was measured after the loose mirror. Upon changing to the second mirror, a mirror with more reflectivity, the power in the cavity increased. This is because the lifetime of the photons inside the cavity is longer leading to a larger population. Another notable change is the increase of the throughput of the permanent mirror for the second loose mirror. The increased power inside the cavity combined with the same reflectivity of the permanent mirror leads to more transmitted power.

With this we could find the gain per round trip and given by :

$$G = 100 \cdot (1 - R_1 R_2) \tag{44}$$

for the first case we have that the gain per round trip is 1.896% now if we have a higher reflectivity mirror as we do in the second case we would expect to have a higher gain. For

the second arrangement of mirrors we have a gain of 0.30%. Now if we are interested in the medium gain this can be calculated from the distributed loss and the mirrors reflectively as follows:

$$MG = \alpha_s + \frac{10}{2d} \cdot \log_{10}\left(\frac{1}{R_1 R_2}\right) \tag{45}$$

In our experimental setup we measured a cavity length of $d = 0.52\ m$ and we are going to assume that there is no distributed loss $\alpha_s = 0$ thus , $GM = 0.0087\frac{dB}{m}$ this shows us that we have an actual gain over the loss of the cavity thus giving us a lasing effect in the cavity produce a total gain in the system. Finally, the alignment laser was amplified using the experimental cavity. Powers P_{ac} and P_{In} when the pumping was active and inactive respectively was measured giving the small signal gain G_{ss} as

$$P_{ac} = 27.6\ mW\ ,\ P_{In} = 23.6 \rightarrow G_{ss} = \frac{P_{ac}}{P_{In}} = 1.17. \tag{46}$$

This shows that there is a total effect of amplification when trying to amplifying laser with the gain medium implemented. This has multiple applications, one is in the telecommunications industry when trying to amplify long haul messages.

6 Conclusions

In this lab we went over the fundamental components and functions of a laser, going through a detailed analysis of its components. We went over the gain medium, the mirrors and the cavity itself, we learned form the implementation that there are critical parameters that determine the power of the laser. We also studied the fundamental modes that exist inside the cavity and were able to isolate and observe a few longitudinal modes in the far field. We also went over a basic amplification of an incoming laser, using the existing cavity. In general we studied all the basic components and also found how to optimize the output power of the laser by altering the cavity length and the reflectivity of the mirrors. This lab exercise has given a general experimental comprehension of what the main challenges of trying to create a laser are and how to execute basic fundamental measurements of quality and calibration.

References

[1] B.E.A. Saleh and M.C. Teich. *Fundamentals of Photonics*. John Wiley Sons, Inc., Hoboken, 2007.

[2] Lab instructions: Helium-neon laser.