Optical Interferometry

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In this experiment we used a Michelson interferometer to study different sets of interference patterns, we we also interested in using interference to study double exposure Holograms, we finished by implemented a setup that allowed us measure small displacements by looking at the scattered speckles from a bridge. Are interest in interferometry used as a tool to measure the cavity length of a laser or determining higher order wavelengths from a single source of light. With a white light calibrated interferometer we also measured the refractive index for an unknown piece of glass by knowing the thickness of this piece. Finally, we studied the speckles movement for a deforming bridge where we could make millimetric measurements with out any mechanical measurement.

I. INTRODUCTION

Interference results from the superposition principle of several waves with the same wavelength. This phenomenon can be used in different ways for measuring small displacements (of the order of the wavelength of the source). Depending on which movements one is interested about, it is wise to adopt one particular method. Interferometry for example is commonly used for measuring tiny displacements in the same direction as the observation direction. One of the most famous application is the interferometer LIGO that is able to measure gravitational waves (extremely difficult to detect otherwise).

However, when the object moves in a perpendicular plane to the incoming rays, it is more judicious to use speckles photography instead. Holography is another method for studying deformations or more generally any physical modifications of three-dimensional diffuse objects that is once again based on the interference principle. Indeed, the image resulting from the superposition of two holograms of the same object recorded at different times, commonly called double-exposed holograms, may show us an interference pattern that will inform us about any deformations endured by the object during this period of time.

II. THEORETICAL BACKGROUND

Interference is an optical phenomenon that may be observed when two waves at least propagate in the same space at the same time. The total wavefunction resulting from this superposition is the sum of all the wavefunctions. Thanks to this superposition principle, one is able to explain the reason why the superposition of two lights can lead to an alternation of dark and and bright spots for example. Indeed, in the particular situation of two monochromatic waves of complex amplitudes respectively $U_1(r)$ and $U_2(r)$, their superposition leads to a wave of complex amplitude $U(r)$ with $U(r) = U_1(r) + U_2(r)$. As the intensity is defined as $I = |U(r)|^2$, we get :

$$I = |U_1(r) + U_2(r)|^2 = I_1 + I_2 + 2\sqrt{I_1 + I_2}cos(\phi) \quad (1)$$

where I_1 and I_2 are respectively the intensities of the two waves and ϕ the difference between their phases. Since ϕ may be a space function, the total intensity resulting from the superposition of the two waves may also be a space function.

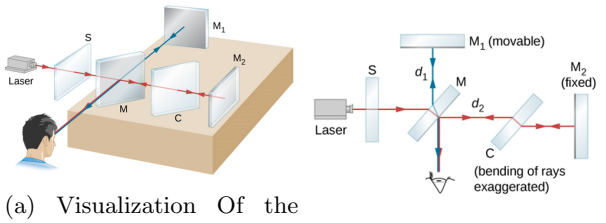
The interference phenomenon is not specific to light, it can be observed for all types of waves. Nowadays, it is largely used in different fields. Ear protectors for example can decrease dramatically external sounds by superposing similar sound waves to the ones issued from the outside but with a phase difference of π so that they interfere destructively.

A. Interferometry

There are different interferometers but we will only focus on Michelson interferometer to understand their function in general. Commonly, a Michelson interferometer is composed of a source, a beam splitter, two mirrors and a screen to visualize the interference pattern as seen in Figure.1a. Our own interferometer had also two lenses : one was used for increasing the width of the beam and the other one for collimating the rays before them to hit the beamsplitter.

After the incoming wave of complex amplitude U and intensity I_0 first hits a beamsplitter, it is divided into two rays of same intensity $I_0/2$: a reflected one (represented in blue in Figure.1b) and a transmitted one (that appears in red in Figure.1b) of respective complex amplitudes U_r and U_t . Each of these two waves then runs a certain distance L and L' (corresponding to the length of the arm in which it propagates) before being reflected by a mirror. After this reflection, each ray comes back to the beamsplitter and is again separated into two rays : a reflected one (U_{rr} for the

1st reflected ray, U_{tr} for the 1st transmitted ray) and a transmitted one (U_{rt} for the 1st reflected ray, U_{tt} for the 1st transmitted ray) so that at the end, we can observe two interference patterns on two different screens : one occurs because of the superposition of U_{rt} and U_{tr} and the other one results from the superposition of U_{rr} and U_{tt}. On Figure.1b, the superposition of the waves of complex amplitudes U_{rr} and U_{tt} is not represented.



(a) Visualization Of the interferometer used for sodium and white light sources

(b) General setup for a Michelson interferometer

FIG. 1: Experimental setups used to detect interference patterns[4]

Moreover, if one rotates one of the two mirrors, the interference pattern will varies. If we rotate for example the Mirror 2 around a perpendicular axis to the direction of propagation, then the space between the fringes will change. One can visually understand how the interference pattern would be affected by looking at Figure2.

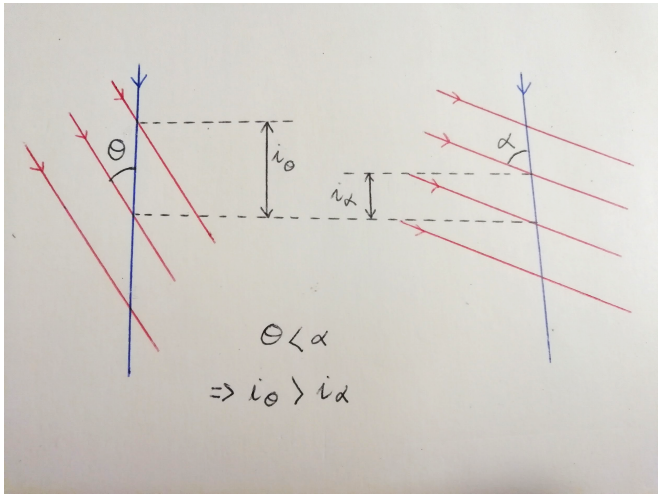


FIG. 2: Drawing that illustrates the influence of the angle between two waves that interfere on the space between two fringes : the bigger is the angle the smaller is the space between two fringes

It is also possible to demonstrate this mathematically. To simplify the calculations, we will consider two plane waves of same intensity I_o propagating in the z direction : one travels with an angle θ with the respect to z-axis U_2 in the xz-plane and the other one is parallel to the z-axis U_1 expressed as[3]:

$$U_1 = \sqrt{I_o}e^{-ikz} \tag{2}$$

$$U_2 = \sqrt{I_o}e^{-ik(z\cos(\theta)+z\sin(\theta))} \tag{3}$$

thus, giving an intensity

$$I = |U|^2 = 2I_0(1+\cos(kx\sin\theta)) \tag{4}$$

This shows us that the space between fringes depends on the angle between the different waves that interfere.

To understand a little deeper the interference pattern that one may observe with a white lamp for example, it is then more convenient to consider the optical waves as spherical ones under the paraxial approximation to simplify our calculation. The theoretical model of these interference patterns can be done by considering that S_1 and S_2 are two sources of spherical waves of same wavelength λ and of respectively complex amplitudes U_1 and U_2 that propagate along the z-axis and $d = 2\|L - L'|$ with L and L' the lengths of the two arms of the spectrometer giving $z_2 = z,\ z_1 = z + d$. Under the paraxial approximation, we have:

$$U_1 = \frac{A_0}{z+d}e^{-ik(z+d)}e^{-ik\frac{x^2+y^2}{2(z+d)}} \tag{5}$$

$$U_2 = \frac{A_0}{z}e^{-ikz}e^{-ik\frac{x^2+y^2}{2z}} \tag{6}$$

where A_0 is a positive real constant. With this we can find the resulting intensity I given by:

$$I = |U_1 + U_2|^2 = \tag{7}$$

$$= \frac{A_0^2}{z^2} + \frac{A_0^2}{(z+d)^2} + 2\frac{A_0^2}{z(z+d)}\cos\left(kd\left(1 - \frac{x^2+y^2}{2z(z+d)}\right)\right) \tag{8}$$

where $k = 2\pi/\lambda$ is the wavenumber

At this point, we can understand that the interference pattern will look like concentric circles of respective radius $\rho = \sqrt{x^2+y^2}$ and the distance between them will get smaller and smaller since $I \approx \cos\rho^2$ as shown in Figure.3. Furthermore, we can also see that the interference pattern depends on the distance d between the two sources, that is to say the difference of length between the two arms of the spectrometer or more precisely the difference of optical path length between the two sources (which can change without removing any of the mirrors but just by adding a glass plate of refractive index n in one of the two arms).

In the particular situation of where the path length in the two arms is the same, the intensity I is constant and equal to $4(A_0^2)/z^2$. Otherwise, we should observe a big bright spot on the screen. Experimentally, we will also

observe fringes around this spot since for the calculation we did some approximations. Nevertheless, the point is that this specific configuration of the interferometer can easily be found, one just has to move one of the mirrors until the middle spot on the screen is the biggest as possible.

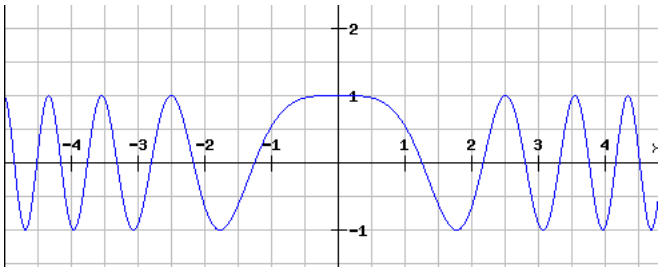


FIG. 3: Plot of the function $y = \cos x^2$

Furthermore, the interference pattern can be damped. Since only waves that have the same wavelength λ can interfere, we might see different interference patterns which overlap constructively or destructively given that there are several wavelengths. As the space between fringes for an interference pattern depends on λ (see 4), the superposition of these different patterns on the screen may reduce the contrast between the fringes. If the two interference patterns are well superposed with each other as shown in the left drawing of Figure.4 the contrast of the resulting image will be high. On the contrary, if the two interference patterns are complementary, i.e one bright fringe of the first interference pattern overlaps a dark fringe of the second interference pattern as shown in the right drawing of Figure.4, the final visibility will be low and on will not be able to distinguish any fringes.

Even a laser may have different wavelengths due to different longitudinal modes in its cavity that is why the visibility of the image obtained with a laser will depend on the relative positions of the two mirrors. But this "drawback" can be used to find the cavity length of a laser for example since the visibility is the highest for path lengths that are multiples of the double cavity length. Thus, in the case of the Michelson interferometer, *if we remove one of the two arms of a length equal to the cavity length of the laser, we will go from a maximum (resp. a minimum) to another.*

In the specific case that the source is a white light, the interference pattern should look like a rainbow since the different interference patterns issued from the different wavelengths will not have the same space between the fringes given that $I \approx \cos(2\pi d\lambda)$ (see equation 8). However, if the mirrors are not at the same distance from the beam splitter, one will only observe a bright spot since a light source has a very small coherence length.

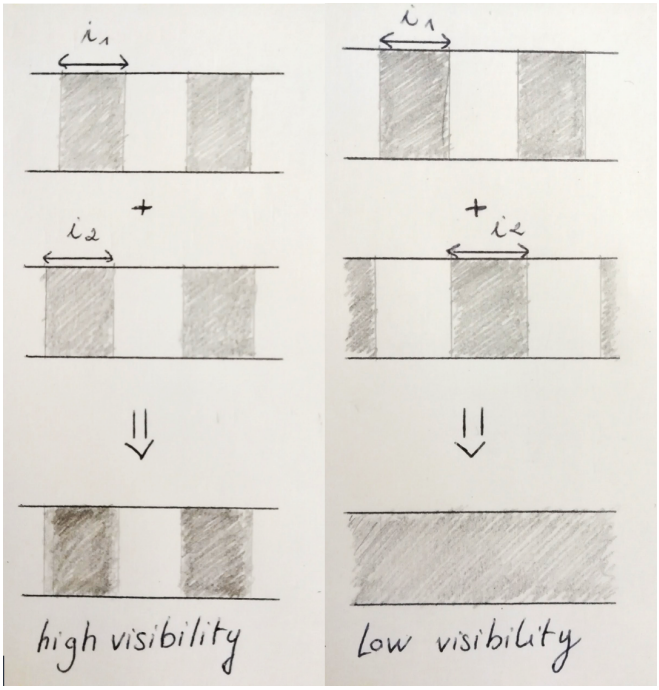


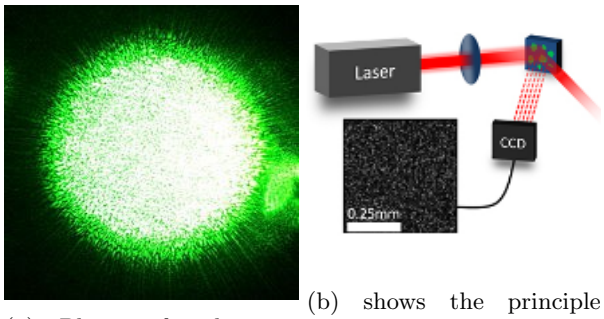
FIG. 4: Drawing that explains why the visibility of the interference pattern can vary if there are different longitudinal modes

B. Speckle correlation

When we illuminate a microscopic non smooth surface with a coherent light, the light spot seems to be grainy with bright and dark dots. These little shining spots are so called subjective speckles (see Figure.5a). This phenomenon is due to the interference of the diverse rays coming from the source and reflected by the object. As its surface is irregular, the reflected rays can interfere with each other according to a random pattern Figure.5b. Therefore, two different people will not see the same image since the interference pattern that we can observe depends on where our eyes are located. Even a single person will see the pattern changing if they change their point of view. But not only speckles pattern change according to one's position but even their sizes can vary if we look through a small hole or a lens.

In opposition to subjective speckles, objective speckles result from the projection on a screen of incoming rays that have been reflected by an irregular surface. As the speckles are imaged on a screen, everyone will see the same pattern. Speckle patterns can be used for measuring displacements that are perpendicular to the viewing direction. For example, if we fix a camera and then take pictures of the speckle pattern on a rough object before and after its tiny displacement, we can find the distance and the direction that the object moved. Indeed, as the displacement of the object is small, the

speckle pattern recorded by the camera before and after the movement of translation will not change that much, it will only be translated in the camera's plane. After processing the data we will be able to associate this shifting pattern to a vector field that will inform us about the global movement of the object.



(a) Photo of subjective speckles obtained with a green laser

(b) shows the principle of formation of subjective speckles

FIG. 5: Speckles example and experimental setup used for measurement execution[1]

From the data analysis one can find the average norm of the displacement r in pixels and since we know the real dimensions (x, y) of the object in the image plane and the camera has $m * n$ pixels, one is able to find the real displacement of the object applying the relation:

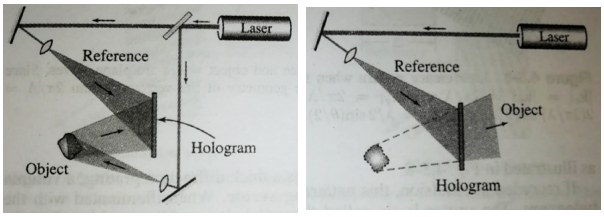
$$l_x = r\frac{x}{m},\ l_y = r\frac{y}{n}. \tag{9}$$

However the final result highly depends on the accuracy of the camera. One has to consider before any manipulations that the speckle size will not restrict the accuracy of the measurement and that their displacements will be recorded on the pictures. Thereby, it seemed us reasonable to image a speckle on about 6 pixels. If a speckle was imaged in a smaller spot than a pixel, we won't be sure to notice its displacement after a tiny movement of the object for example.

C. Holographic interferometry

A hologram is the interference pattern recorded on a transparent material between a reference wave Ur and an object wave U_o. The two waves are actually issued from the same source (they can therefore interfere with each other). After being divided by a beamsplitter for example, one of them (U_o) will be reflected on an object before interfering with the other one (U_r). In this way, what is recorded on the transparent plate is an interferometric image that will reveal more or less bright areas when illuminated by the reference wave as shown in Figure.6b. The brightest areas will be those in which the light from the object and the reference beam are in phase.

This brightness will also depend on the amplitude of the light coming from the object. For the reconstruction of the object, the same reference wave U_r enlightens the hologram giving us the observer a virtual image of the object, the basic setup is shown in Figure.6a.



(a) Recording

(b) Reconstruction

FIG. 6: shows the principle or construction and lecture of a hologram

Contrary to photography which is only sensible to variations of intensity, a hologram carries information about the phase of the object wave. Holography can be used for detecting changes or deformations (of the order of the wavelength) of a three-dimensional diffuse object.

One can superimpose the hologram of the object before its deformation on the light reflected by the object after its deformation. Another option consists in superimposing two holograms of the same object before and after its deformation and then enlightening the plate containing the two holograms. In both of the situations, one will observe interference fringes on the final image. It is then possible to have an idea of the deformation imposed to the object by studying the interference pattern of the final image

III. EXPERIMENTAL METHOD

A. Interferometry

1. *Visibility curve for a He-Ne laser*

In this experiment, the source was a Helium-Neon (He-Ne) laser. First, we put the two mirrors at equal distance from the beam splitter and aligned them by moving them one by one and making the back reflected beam pass exactly the same way in cam threw.

Then, by moving one of the two mirrors we created a phase difference between the rays coming back from each mirror (which were on phase before) and we were then able to observe a displacement of the fringes in

the interference pattern as discussed in the previews section. Another point of interest in this exercise was the variations of visibility V of the image according to the difference of length d between the two arms. For some positions of the moving mirror along the x-axis, the visibility was very low in opposition to some other positions for which the visibility was high as discussed in the previews section and shown in the Figure.4. We measured the distance between the mirror and the beam splitter for each position of interest (we created a scale of visibility between 0 and 1 in order to rank them and give us an idea of the evolution of the visibility according to the position of the moving mirror).

With those data we were then able to make a plot as a function of the path length difference. This difference in visibility maximum gives us information on the length of the optical cavity inside the laser. Where we can show that it corresponds to approximately the same length as the laser cavity, this length would be more accurate if we had a very high resolution measurement.

2. *Interference in sodium (Na) light*

In this experiment we used once again the Michelson interferometer method but this time with a diffuse source : a sodium light. Like in the other experiment we could observe fringes due to the interference of the different wavelengths. To find back the position for which the two arms had the same length, we moved one of the mirrors while looking at the fringe pattern until the distance between the fringes were the biggest. We also observed a scale of visibility depending on where we put the moving mirror since the source was composed of two different wavelengths. If one knows one of the wavelength then we can calculate a relation between them since one wavelength dose one more revolution than the other due to the difference in frequency the shorter wavelength can fit more oscillations in the same cavity or distance giving us a relation between the wave lengths.

$$\begin{cases} if\ \lambda_2 < \lambda_1 \\ \rightarrow 2\Delta L = n\lambda_1\ ,\ 2\Delta L = (n+1)\lambda_2 \\ \rightarrow \lambda_2 = \frac{2\Delta L \lambda_1}{2\Delta L + \lambda_1} \end{cases} \tag{10}$$

3. *Interference in white light*

This was the same experiment as int he previews subsection, but now we replaced the sodium light with a white light. We put the mirror where the difference of path length was near zero like in the last experiment since the fringes can only be seen in this position, otherwise the different interference patterns will overlap. Then we moved very gently the mirror to obtain coloured fringes. When aligned we tried to calculate the refractive index n of a glass plate of thickness t known. We introduced a glass plane between the movable mirror and the beam splitter and re adjust the length to observe again the coloured fringes. To achieve this we had to bring closer the moving mirror to the beamsplitter a distance d since the wavelength changed in the glass $\lambda = \lambda_0/n$ and so altering the optical path length . The optical path length in the moving arm was given by :

$$OPL = 2(L-t)n + 2tn$$

with the L being the length both arms from the beamsplitter and d the total change in the optical length, giving us a way to write:

$$d = \frac{OPL}{2} - L\ ,\ d = t(n-1) \rightarrow n = \frac{d}{t} + 1 \tag{11}$$

B. Holographic interferometry

In this part of the laboratory exercise, we focused on a holographic plate resulting from the superposition of two holograms of the same object : one was recorded before the object was being deformed and the other one after so that we could observe an interference pattern shown in Figure.7

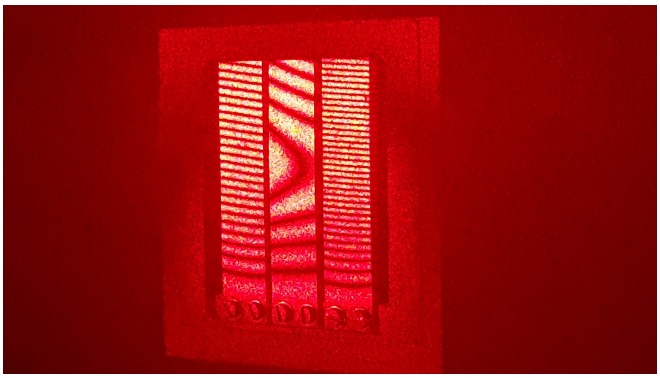


FIG. 7: Hologram studied in the laboratory. It resulted from the overlap of two holograms of the same object before and after its deformation. The interference patterns observed on each bar can reveal us the deformation imposed to the object

C. Speckle correlation

In the laboratory exercise, we enlightened a wooden bridge with a homogeneous laser beam whose light had been redirected thanks to a couple of mirrors. A camera CCD connected to a computer was used for taking pictures of the speckle patterns Figure.5b. The objective was to try to measure the displacement of the bridge after placing different weights.

Before taking any pictures, the camera was focused on the bridge by using a grid paper to be sure that the

focus was the best as possible. Then, we change the focus of the camera in such a way that a speckle was at least bigger than a couple of pixels. Afterwards, we put different weights on the wooden bridge (0g, 200g, 400g, 600g) and took pictures every time. The displacements endured by the bridge each time a new weight was load lead to different images due to the change of position of the speckles.

IV. RESULT & ANALYSIS

A. Speckle correlation

Form the images recollected we were able to find the movement of the bridge from the deformation given by the weights. With this images and computational analysis we are able to capture the displacement in micrometers for all the different weights as shown in Figure.8 as a contour map, and as a vector displacement of the speckles in Figure.9.

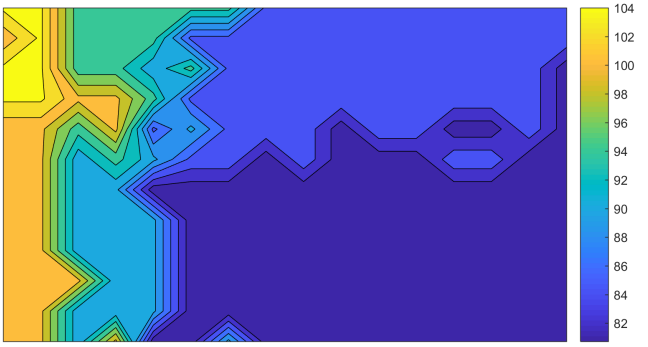


FIG. 8: Level map of the displacement of the bridge under stress from a set of different weights. Where the color bar is in micrometers and the maximum translation was 104 μm. The plot shows the displacement over all the images taken in a arbitrary scale on the axis

From the data recollected we can conclude that the over all tendency was a displacement directed to the bottom left bridge and the most intense displacement was in the top the top left with a magnitude of 104 μm movement.

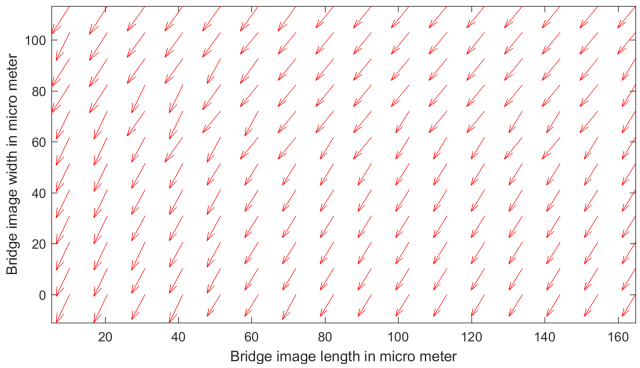


FIG. 9: Vector plot of the captured displacement of the speckles over all the images recollected where we ca see there is an overall tendency to be deformed to the bottom left corner

This makes fits within the expected model since the weights used to deform the bridge had a maximum load of 600 g producing a small deformation which dose not change the fiscal integrity of the system, making it hard to observe a fiscal deformation by eye. Thus, making this result an accurate description of what was observe at the time of executing the measurement. This lets us see the strength of this technique since we can measure small fluctuation with a relative simple set up and without the need of any mechanical parts.

B. Interferometry

1. *Visibility curve for a He-Ne laser*

For this section we used two different Michelson interferometers, with one we observed the visibility curve for a He-Ne laser and with the second we looked at the interference pattern of a sodium and white light source. First we registered the visibility curve for the He-Ne light source from this the distance between maximum visibility was measured to be $\Delta L = 33.6\ cm$ giving us insight on the cavity length inside the laser. Looking at the average lengths of a cavity for a He-Ne laser we found that its usual length is between 15 cm to 50 cm as shown in Reference[6]. With this we can see that the value measured fits perfectly in the range of normal cavity lengths, also it is consistent with the size of the laser used in the experiment.

2. *Interference in sodium (Na) light*

With the second interferometer we studied the interference pattern for sodium and white light. For the sodium source we scanned for minimum visibility with the movable arm obtaining the data given in Table.I giving us average distance between minimum visibility

of $\Delta L = 0.29$cm

Position [mm]	Difference [mm]
11.22	-
11.5	0.28
11.81	0.31
12.10	0.29

TABLE I: Data collected for distance between minimum visibility for interference in sodium source giving an average distance of $\Delta L = 0.29$mm

With this we can find information of one set of wave lengths using the Equation.10 given that we have the difference in length and assuming that we know the larger wavelength 589.6nm we get that the second wavelength should be 589.0012nm which has an error of 0.0012nm with respect to the known value of the emission wavelengths of this lamp.

3. Interference in white light

For this set up we calibrated the system at a position where we could observe interference with the white light source, after this we placed the piece of glass in the optical path of the light. From this position moved the mirror until we saw a interference pattern in the piece of glass. With these well defined lengths we could use Equation.11 to find the refractive index of the piece of glass. The thickness of the glass was $t = 1.03$mm, substituting the known values we get a refractive index of $n = 1.5534$. This refractive index is within the typical values for glass which reinforces the versatility of this technique at the time of measuring small fluctuations in path lengths and what can be done with its implementation.

C. Holographic interferometry

This section was directed to understanding what was the effect measured in the hologram shown in Figure.7. As it was discussed this is two holograms that overlapped where we cant distinguish in which order they were taken. One of the holograms was taken with the system in a relaxed state and the other one was taken when the system was being deformed by different sets of force being applied. First, we can tell that the right side is more deformed since the spacing in the interference pattern is smaller giving a larger angle between the interfering waves and there is a change in the spacing due to a rotation on there on axis. For the left column we can say that there is a different amount of separation for the top and the bottom giving us the effect of a change in the spacing in the interference pattern. Both of these two columns had probably been rotated around their axis since the fringes are slightly leaning. The last column we looked at was the middle one since this one gives a more complicated interference pattern. What we could deduce was that this system must be getting deformed in two dimensions giving some curvature, thus producing this alternating interference pattern.

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