Hand in problems Week 1

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Exercise & Solution

Problem 1

Consider a fiber where the difference in refractive indices is $\Delta n = 0.0075$ and the core radius $4\ \mu m$. What is the maximum n_1 such that the fiber behaves like a single mode fiber in the whole C-band?

First lets note that the C-band corresponds to the wavelength range of $1530\ nm - 1565\ nm$ to design a fiber that behaves as a single mode for this wavelength range we have to consider the cutoff wavelength. The cutoff wavelength of a fiber λ_{cf} is the smallest wavelength for which a fibre is single mode, this wavelength can be determined from the cutoff condition for single mode fibres shown in Eq.1 as shown in Ref [1][2]. With this we can proceed to determine the fiscal properties of are desired fibre.

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} \leq 2.405 \tag{1}$$

Taking into account the information given we can determine V for the given difference in refractive index as follows :

$$\Delta n = n_1 - n_2 = 0.0075 \rightarrow V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - (n_1 - 0.0075)^2}$$

Now since we want to find the maximum n_1 from the cutoff condition we can select the upper limit of Eq.1 where we select then the cutoff wavelength, solving for n_1:

$$\frac{2\pi}{\lambda_{cf}} a \sqrt{n_1^2 - (n_1 - 0.0075)^2} = 2.405$$

$$n_1^2 - (n_1^2 - 0.015 n_1 + 0.00005625) = \frac{\lambda_{cf}^2}{4\pi^2 a^2} 5.784025$$

Giving an expression for the refractive index in terms of the cutoff wavelength and the core radius a:

$$n_1 = \frac{1}{0.015}\left(\frac{\lambda_{cf}^2}{4\pi^2 a^2}5.784025 + 0.00005625\right) \tag{2}$$

With this we can determine the inner cores refractive index of are desired fibre knowing that $a = 4\mu m$, $\lambda_{cf} = 1530nm$ since this is the smallest wavelength of the band the is a cutoff wavelength following the definition given previously. For this conditions we find that the inner core refractive index is $n_1 = 1.43278$

Problem 2

In Problem 2.1, Equation (2.2) is verified. In the text on page 54 the fractional refractive index $\Delta = \frac{n_1 - n_2}{n_1}$is defined. There, and later in the text, the following approximations are used,

$$\theta_{max}^0 = \sin^{-1}\frac{\sqrt{n_1^2 - n_2^2}}{n_0} \approx \sin^{-1}\frac{n_1\sqrt{2\Delta}}{n_0} \approx \frac{n_1\sqrt{2\Delta}}{n_0}$$

Verify and motivate the approximations.

For this problem we want to find what is the condition of the coupling angle to achieve total internal reflection inside the fiber. The scheme of the fiber is shown in Fig,1 where we we use the geometry of the problem to find a relation between the internal angle.

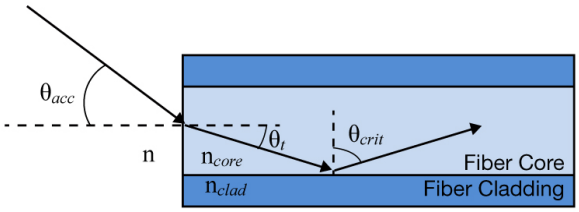


Figure 1: Optical fiber scheme used threw out the problem

The critical physical principal used threw out this problem is the Snell's law, form which we can find the critical angle where we have internal reflection. Total internal internal reflection is achieved when the angle of the transmitted ray is larger than $\frac{\pi}{2}$, so for any angle of incidence smaller than the critical angle total internal reflection will be produced.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \xrightarrow{\theta_2 = \frac{\pi}{2}} \theta_{critt} = \sin^{-1}\frac{n_2}{n_1} \tag{3}$$

Following the geometry in Fig.1 we can write,

$$n \sin\theta_{acc} = n_{core}\sin\theta_t \rightarrow \theta_t = 90 - \theta_{critt}$$

Now we can also find the condition of the critical angle for are specific case and write it in terms of the incident angle with he fiber it self.

$$\begin{aligned}\theta_{acc} &= \sin^{-1}\left(\frac{n_{core}}{n}\sin\theta_t\right) = \sin^{-1}\left(\frac{n_{core}}{n}\sin\left(90-\theta_{critt}\right)\right)\\ &= \sin^{-1}\left(\frac{n_{core}}{n}\sin\left(90-\sin^{-1}\frac{n_{clad}}{n_{core}}\right)\right)\end{aligned}$$

Its useful to remember some basic trigonometric properties,

$$\sin\left(\theta-90\right) = \cos\theta\ ,\ \cos\theta = \sqrt{1-\sin^2\theta} \tag{4}$$

First to clear up the notation and follow the one used in the problem we will change the refractive index as follows $n = n_0$, $n_{core} = n_1$ and $n_{clad} = n_2$. And plugging in the trigonometrical identities shown we get:

$$\theta_{acc} = \sin^{-1}\left(\frac{n_1}{n_0}\sqrt{1-\sin\left(\sin^{-1}\frac{n_2}{n_1}\right)^2}\right) = \sin^{-1}\left(\frac{\sqrt{n_1^2-n_2^2}}{n_0}\right) \tag{5}$$

Which is precisely the acceptance angle we expected, now we can make some approximations. First we know that experimentally the refractive index of the core and the cladding are very similar such that we can do a Taylor expansion of n_2 around n_1. We only take the first two terms,

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) \tag{6}$$

now the function we want to approximate is the inner factor of the square root:

$$f(x) = n_1^2 - x^2 \rightarrow f(x) \approx f(n_1) + f'(n_1)(x-n_1) \approx 2n_1(n_1-x)$$

We are evaluating the approximation around n_2 so we replace in the expression found in Eq.5 giving us :

$$\theta_{acc} = \sin^{-1}\left(\frac{\sqrt{n_1^2-n_2^2}}{n_0}\right) \approx \sin^{-1}\left(\frac{\sqrt{2n_1(n_1-n_2)}}{n_0}\right) \tag{7}$$

Which can be neatly written if we define the factor $\Delta = \frac{n_1-n_2}{n_1}$. Finally we can also consider small angles and take the first term of the Taylor series of the *sin* function $sin\theta \approx \theta$

$$\theta_{acc} \approx \sin^{-1}\left(\frac{n_1\sqrt{2\Delta}}{n_0}\right) \xrightarrow{\theta_{acc}\ll} \theta_{acc} \approx \frac{n_1\sqrt{2\Delta}}{n_0} \tag{8}$$

References

[1] R. Ramaswami, K. Sivarajan and G. Sasaki: Optical Networks: A Practical Perspective, 3rd Ed. ISBN: 978-0-12-374092-2

[2] G. P. Agrawal: Fiber-optic communication system, 4th Ed. ISBN:9780470505113