Hand in problems Week 2

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Exercise & Solution

Problem 1

Consider the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + V(\mathbf{r})\Psi(\mathbf{r}) = \mathbf{E}\Psi(\mathbf{r}) \tag{1}$$

- **Write down the one-dimensional version in cartesian coordinates.**
- **For a free electron (V=0) of mass m_0. Use a trial solution of the form $\Psi(x) \propto e^{-ikx}$ determine the value of $E_x(p, m_0)$**

To consider a one dimensional wave equation we can just take a single element of the three dimensional representation of Eq.1. this can be simply done by changing $\nabla^2 \to \nabla_x^2$ and $r \to x$. With this we can rewrite Eq.1 as :

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(\mathbf{x}) + V(\mathbf{x})\Psi(\mathbf{x}) = \mathbf{E}\Psi(\mathbf{x}) \tag{2}$$

Now if we say that there is no potential acting on the particle or considering as it was traveling as a free, we can set the potential V to zero. And we can make an assumption on the wave function by setting it to $\Psi(x) = Ce^{-ikx}$ where is a constant to make sure the function is normalized.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(\mathbf{x}) = \mathbf{E}\Psi(\mathbf{x})$$

$$P(x) = |\Psi(x)|^2 \to \int_\infty^\infty |\Psi(x)|^2 = 1 \to \int_\infty^\infty |C|^2 = 1$$

We can find a normalization condition by understanding the relation between the wave function and the probability, thus when we add all the probabilities the should be one. Intoducing are wave function we can find the energy given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}Ce^{-ikx} = \mathbf{E}Ce^{-ikx} \tag{3}$$

$$-\frac{\hbar^2}{2m}C(-ik)^2e^{-ikx} = \mathbf{E}Ce^{-ikx} \rightarrow -\frac{\hbar^2}{2m}(-ik)^2 = \mathbf{E} \tag{4}$$

$$\mathbf{E} = \frac{\hbar^2k^2}{2m} \rightarrow \mathbf{E}(p, m_0) = \frac{p^2}{2m_0} \tag{5}$$

Problem 2

Consider an InGaAsP laser diode with refractive index n=3.5 and a cavity length of d= 400 micrometers.

- **Calculate the frequency spacing ν_F in GHz of the resonator modes (longitudinal modes).**
- **For a central wavelength of 1300 nm, calculate the wavelength spacing between the modes in nanometers.**
- **If the frequency spectral width is $\Delta\nu = 1.2\ THz$ calculate the wavelength spectral width in nanometers.**
- **Determine the number of longitudinal modes emitted in this spectral width.**
- **If we want to reduce the number of modes emitted to one, calculate the required cavity length.**

To find the frequency spacing of a cavity can be done by using the definition given in [3], from this we can just substitute with are cavity parameters as follows:

$$\nu_F = \frac{c}{2d} = \frac{c_0}{2nd} \tag{6}$$

$$\nu_F = \frac{3 \cdot 10^8 \frac{m}{s}}{2 \cdot 3.5 \cdot 400 \cdot 10^{-6}m} = 107.07\ GHz \tag{7}$$

To find the wavelength spacing around a central wavelength of 1300 nm is staring forward knowing the frequency spacing:

$$\nu_F = \frac{c}{\lambda^2}\Delta\lambda \rightarrow \Delta\lambda = \nu_F\frac{\lambda^2 n}{c_0} = 2.1125\ nm \tag{8}$$

For the case that the frequency spectral width of $\Delta\nu = 1.2THz$ we get a total wavelength spectral width of:

$$\Delta\nu = \frac{c_0}{n\lambda^2}\Delta\lambda \rightarrow \Delta\lambda = \frac{n\lambda^2}{c_0}\Delta\nu = 23.6764\ nm \tag{9}$$

We can find the number of modes in this spectral width by just divide are total spectral width by the seperation we have giving us a number if modes N :

$$N = \frac{\Delta\nu}{\nu_F} = \frac{1.2\ THz}{107.07\ GHz} = 11.2078 \approx 11 \tag{10}$$

If we want a single mode (N=1) we need to change the cavity length to a smaller one and we can find this length by using the previous relation:

$$N = \frac{\Delta\nu}{\nu_F} = \frac{2\ nd\Delta\nu}{c_0} \rightarrow d = N\frac{c_0}{2\ n\Delta\nu} = 35.69\ \mu m \tag{11}$$

References

[1] R. Ramaswami, K. Sivarajan and G. Sasaki: Optical Networks: A Practical Perspective, 3rd Ed. ISBN: 978-0-12-374092-2

[2] G. P. Agrawal: Fiber-optic communication system, 4th Ed. ISBN:9780470505113

[3] Fundamental of Photonics, second edition, B. E. A. Saleh and M. C. Teich