Hand in problems Week 3

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Exercise & Solution

A communication system is transmitting at a data rate of 10 Gbps by using NRZ OOK modulation with bandwidth efficience ρ= 0.4 bps/Hz. The optical output powers at the laser are 1 mW for a transmitted 1 and 0.05 mW for a transmitted 0, and the fiber has the an attenuation of 0.25 dB/km. At the receiver the photodetector has the responsivity R = 1.25 A/W, and the subsequent electrical circuit has a load resistance of 100 Ω. Assume that the system is working at room temperature.

- **Plot the bit error probability as a function of the length in km. What is the limit on the distance if the bit error rate must not exceed 10^{-12}**
- **By complementing the system with an error correcting code (FEC) the reach can be extended. Assume you use a code that has a coding gain of 3 dB, redo problem (a) for this case.**

To find the bit error rate (BER) we first need to use a set of definitions given in Ref.[1]. This analysis is fairly straight forward if we assume the definitions as true and proceed to implement the numerical analysis for our particular case first we need to define what are the values of the photocurrent and the optical power received at the photo detector. The optical power in our case it's given, then it was experimentally determined at the output for a binary bit. Now the photocurrent can be calculated since $I_i = R \cdot P_i$ and we can find the deviation from this value by the given expression [1]:

$$\sigma_i^2 = 2eI_iB_e + \frac{4k_bTB_e}{R_L} \tag{1}$$

With this we can find the threshold current which defines the current at which we will decide if the photon current detected corresponds to a zero or a one was sent, this is shown in Fig.1.

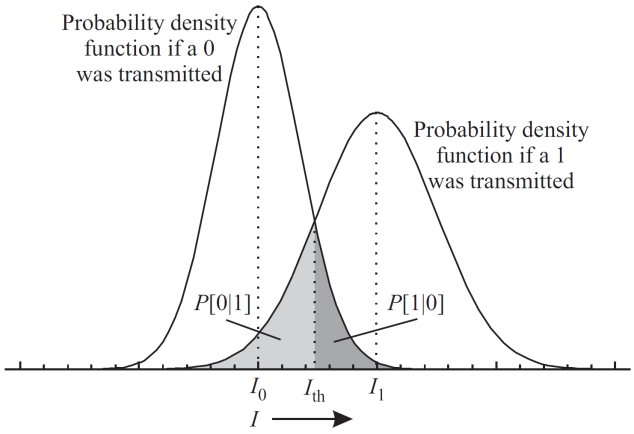


Figure 1: Photocurrent overlap and definition of the threshold current

This threshold current is defined in depending on the expected value of the current and the standard deviation.

$$I_{th} = \frac{\sigma_0 I_1 - \sigma_1 I_0}{I_1 + I_0} \tag{2}$$

Giving us place to define the beat error rate using the well known statistical function Q-function. Using this and the usual statistical problem of over lap of Gaussian functions one can define the bit error rate as is discussed in Ref[1].

$$Q(x) = \int_x^\infty e^{\frac{-y^2}{2}} dy \tag{3}$$

$$BER = Q\left(\frac{I_1 - I_0}{\sigma_0 + \sigma_1}\right) \tag{4}$$

With this definitions we can go ahead and find the bit error rate in the case we consider that we know the optical power in the output and we know what is the attenuation per kilometer in decibels. For this case we only have to take into acount the following of the attenuation power :

$$I_i = \frac{R \cdot P_i}{10^{\frac{\alpha L}{10}}} \tag{5}$$

Where α is the attenuation and L is the distance of the fiber. With all of the previous definitions one can find the distance at which the BER is larger than 10^{-12}. In Fig.2 the BER is plotted against the distance of the fiber and using

software we can find that the longest distance we can have for the cable with out it exiting our set limit is 64 km.

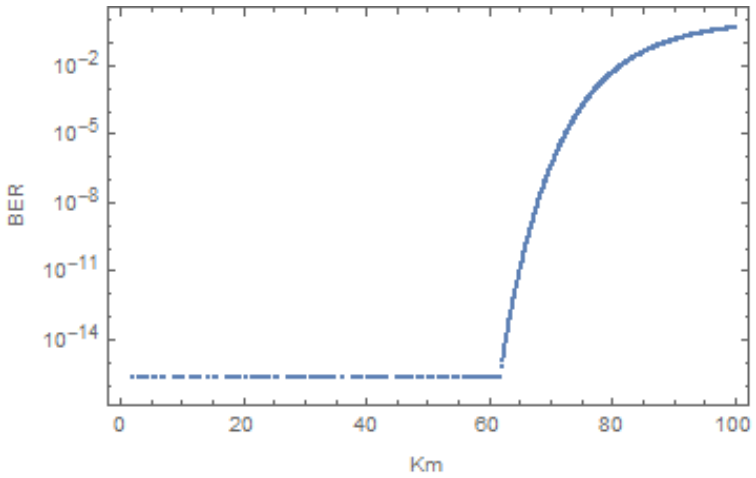


Figure 2: BER plotted logarithmic scale agents the physical distance of a fiber with losses

If we try to improve on this with a code that has a gain of $g = 3\ dB$ we would change our photon current as follows:

$$I_i = R \cdot P_i \frac{10^{\frac{g}{10}}}{10^{\frac{\alpha L}{10}}} \tag{6}$$

Doing the same process we get a relation between the BER and distance as shown in Fig.3. As done previously we can determine that the maximum length before our BER exceeds or preset limit can be determined to be 76 km which is an improvement of around 20% being quite significant advantage.

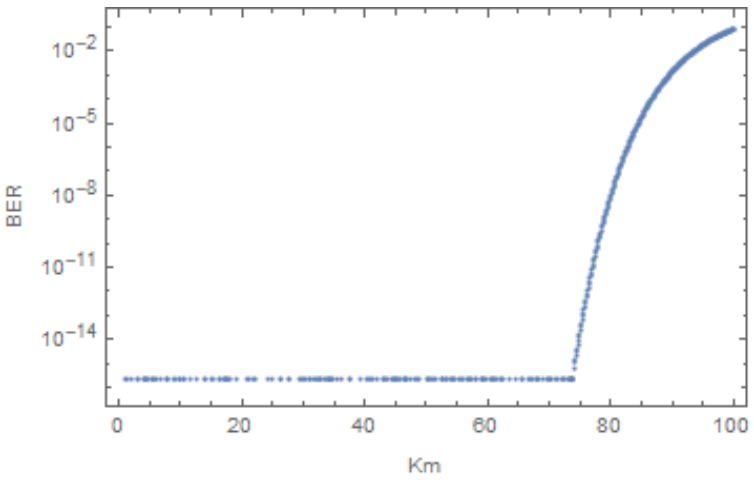


Figure 3: BER plotted logarithmic scale agents the physical distance of a fiber with losses and a gain given by software

References

[1] R. Ramaswami, K. Sivarajan and G. Sasaki: Optical Networks: A Practical Perspective, 3rd Ed. ISBN: 978-0-12-374092-2

[2] G. P. Agrawal: Fiber-optic communication system, 4th Ed. ISBN:9780470505113